Code No: R2021011 **R20** SET - 1

II B. Tech I Semester Regular Examinations, Feb/March - 2022
MATHEMATICS - III
(Com to all branches)

Time: 3 hours Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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1 a) If $f(x,y,z) = 2x^2 + 4xy + 3z$ then find $grad\ f$. [4M]

b) Find the divergence of the vector function $\overline{F} = (x^3 + y^3)\,i + 3xy^2\ j + 3zy^2\ k$. [5M]

c) Calculate the work done in moving a particle in the force field $\overline{F} = 3x^2\ \overline{i} + (2xz - y)\,\overline{j} + z\,\overline{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). [5M]

Or

2 Verify Green's theorem in plane for $\oint_C \left[(3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy\right]$, where C is boundary of the region defined by $y = \sqrt{x}\ and\ y = x^2$. [14M]

3 a) If $L\{f(t)\} = F(s)$ then prove that $L\left\{\int_0^t f(u)du\right\} = \dfrac{F(s)}{s}$. [7M]

b) Find $L^{-1}\left\{\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right\}$. [7M]

Or

4 a) Use transform method to solve the differential equation $\dfrac{d^2x}{dt^2} - 8\dfrac{dx}{dt} + 15x = 9te^t\ \ with\ \ x = 5, \dfrac{dx}{dt} = 10\ \ at\ \ t = 0$. [7M]

b) Find the Laplace Transform of $\left\{\left(\sqrt{t} + \dfrac{1}{\sqrt{t}}\right)^3\right\}$. [7M]

5 a) Find the Fourier series for the function $f(x) = \begin{cases} x & , \ 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$. [7M]

Also deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \cdots\cdots = \dfrac{\pi^2}{8}$.

b) Obtain the Fourier expansion of $f(x) = x\sin x$ as a cosine series in $(0, \pi)$. [7M]

Or

6 a) Find the Fourier cosine integral and Fourier sine integral of $f(x) = e^{-kx}, k > 0$. [7M]

b) Find the Fourier transform of $e^{-a^2x^2}$, $a > 0$. Hence deduce that $e^{-\frac{x^2}{2}}$ is self reciprocal in respect of Fourier transform. [7M]
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7 a) Derive the partial differential equation by eliminating the arbitrary constants from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. [4M]

b) Solve the partial differential equation $\left(x^2 - yz\right)p + \left(y^2 - \mathrm{zx}\right)\mathrm{q} = \mathrm{z}^2 - \mathrm{xy}$. [5M]

c) Solve the partial differential equation $z^2 = 1 + p^2 + q^2$. [5M]

Or

8 a) Form partial differential equation by eliminating the arbitrary functions from $z = f(x) + e^y g(x)$ [4M]

b) Find the general solution of the partial differential equation $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2zx$. [5M]

c) Solve the partial differential equation $p^2 + q^2 = x^2 + y^2$. [5M]

9 a) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. [7M]

b) Solve $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = \sin\left(2x + 3y\right)$. [7M]

Or

10 a) Solve the by the method of separation of variables $4u_x + u_y = 3u \text{ and } u(0, y) = e^{-5y}$. [7M]

b) An insulated rod of length L has its ends A and B maintained at 0^o C and 100^oc respectively until steady state conditions prevail. If B is suddenly reduced to 0^oc and maintained at 0^o C, find the temperature at a distance x from A at time t. [7M]