

II B. Tech I Semester Regular Examinations, Feb/March - 2022
MATHEMATICS - III
 (Com to all branches)

Time: 3 hours

Max. Marks: 70

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

- 1 a) If $f(x, y, z) = 2x^2 + 4xy + 3z$ then find $\text{grad } f$. [4M]
 b) Find the divergence of the vector function $\vec{F} = (x^3 + y^3) \vec{i} + 3xy^2 \vec{j} + 3zy^2 \vec{k}$. [5M]
 c) Calculate the work done in moving a particle in the force field $\vec{F} = 3x^2 \vec{i} + (2xz - y) \vec{j} + z \vec{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). [5M]

Or

- 2 Verify Green's theorem in plane for $\oint_C [(3x^2 - 8y^2) dx + (4y - 6xy) dy]$, where [14M]
 C is boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

- 3 a) If $L\{f(t)\} = F(s)$ then prove that $L\left\{\int_0^t f(u) du\right\} = \frac{F(s)}{s}$. [7M]
 b) Find $L^{-1}\left\{\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right\}$. [7M]

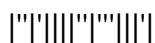
Or

- 4 a) Use transform method to solve the differential equation [7M]
 $\frac{d^2x}{dt^2} - 8\frac{dx}{dt} + 15x = 9te^t$ with $x = 5, \frac{dx}{dt} = 10$ at $t = 0$.
 b) Find the Laplace Transform of $\left\{\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^3\right\}$. [7M]

- 5 a) Find the Fourier series for the function $f(x) = \begin{cases} x & , 0 \leq x \leq \pi \\ 2\pi - x & , \pi \leq x \leq 2\pi \end{cases}$. [7M]
 Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$.
 b) Obtain the Fourier expansion of $f(x) = x \sin x$ as a cosine series in $(0, \pi)$. [7M]

Or

- 6 a) Find the Fourier cosine integral and Fourier sine integral of [7M]
 $f(x) = e^{-kx}, k > 0$.
 b) Find the Fourier transform of $e^{-a^2x^2}, a > 0$. Hence deduce that $e^{-\frac{x^2}{2}}$ is self [7M]
 reciprocal in respect of Fourier transform.



- 7 a) Derive the partial differential equation by eliminating the arbitrary constants [4M]
from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.
- b) Solve the partial differential equation [5M]
equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.
- c) Solve the partial differential equation $z^2 = 1 + p^2 + q^2$. [5M]

Or

- 8 a) Form partial differential equation by eliminating the arbitrary functions from [4M]
 $z = f(x) + e^y g(x)$
- b) Find the general solution of the partial differential equation [5M]
 $(x^2 - y^2 - z^2)p + 2xyq = 2zx$.
- c) Solve the partial differential equation $p^2 + q^2 = x^2 + y^2$. [5M]
- 9 a) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. [7M]
- b) Solve $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = \sin(2x + 3y)$. [7M]

Or

- 10 a) Solve the by the method of separation of variables [7M]
 $4u_x + u_y = 3u$ and $u(0, y) = e^{-5y}$.
- b) An insulated rod of length L has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C , find the temperature at a distance x from A at time t . [7M]

